



MEDTEC SHOOL

Course: **MATHEMATICS**

Year (1st-2nd-3rd-4th-5th-6th): 1

Period (1st-2nd semester – annual): 1

Credits: 10

Objectives

Mathematics is an integrated course that aims to provide the students with the knowledge of the basic concepts of Mathematical Analysis and Linear Algebra. The Calculus session is focused on real functions of one real variable, limits, derivatives, integrals, and their applications to mathematical modeling and optimization. Basics of differential calculus for multivariable functions will also be given. In addition, the modulus provides the basics of the theory of Ordinary Differential Equations, including second order linear equations and their applications to Mechanics. The Linear Algebra session introduces the matrix calculus and the theory of linear systems. Such tools are essential in the construction, analysis and understanding of mathematical models for the description of real world phenomena.

Besides teaching the fundamental mathematical tools for Engineerings and Life Sciences, the course aims to improve the students' critical thinking and problem solving skills.

Lectures and exercises sessions will allow students to acquire the following competences:

- understanding of basic concepts of Mathematical Analysis and their role in the description of real world phenomena (DD1)
- knowledge of basic analytical tools for Engineering and Life Sciences such as limits, derivatives and integrals, linear systems (DD1).
- ability in constructing and analyzing simple mathematical models for the description of real world phenomena via ordinary differential equations
- ability in applying the acquired knowledge to solve simple applied problems of optimization (DD2)
- communication skills: writing and explaining mathematical concepts in a clear and rigorous way (DD4)

Prerequisites

Knowledge of elementary mathematics is required. In particular: Elementary algebra (literal calculus; resolution of first and second degree equations and inequalities, with modules, algebraic, fractional, irrationals).

Trigonometry: basic concepts, elementary trigonometric functions, notable trigonometry identities, trigonometric equations and inequalities. Analytical geometry: Cartesian coordinates, equation of the line, of the circle, outline of conics. Logarithms and their properties, exponential and logarithmic equations and inequalities (*). Graphs of elementary functions: straight lines, parabolas, hyperbolas, elementary trigonometric functions; exponential, logarithm.

Contents

CALCULUS

- Numbers. Integers, Rational and Real numbers: properties and the Axiom of Continuity. Complex numbers: algebraic, trigonometric and exponential representation. Sum and product. De Moivre formula.
- Functions. Real functions of one variable. Domain, graph, properties. Elementary functions. Polynomials x^n as "building blocks". Sigmoidal functions. Trigonometric functions. Exponential functions. Compositions and Inverses. Logarithmic functions.
- Limits. Sequences, limits of sequences. Convergence of monotonic sequences. The Nepero number e . Geometric series. Limit of functions. Limit rules, infinite limits and indeterminate forms. Order of infinities and



infinitesimals. Series and convergence criteria.

- Continuity. Continuity and its consequences. Classes of continuous functions. Basic type of discontinuities. The Extreme Value Theorem and the Intermediate Value Theorem.
- Derivatives. Definition of derivative, geometric and physical interpretation. Rules of derivation (derivatives of elementary functions, chain rule, product rule). Real functions of several variables: partial derivatives, gradient, directional derivatives, differentiability and tangent plane.
- Applications of the derivative. Lagrange Theorem and its consequences. Second derivative and curve sketching. Problem solving: Optimization. (*) Taylor expansion.
- Differential equations. Part I: Linear ODEs of second order. MOOC (with introductory lesson and physical motivations by Andrea Bassi)
- Integration. Indefinite integrals, antiderivatives and techniques of integration: Integration by substitution, by parts, partial fractions and other algebraic techniques.
- Definite integrals and Applications. Riemann-Cauchy sums. Definite integral. The Mean Value Theorem and the Fundamental Theorem of Calculus. Areas and real world applications of integration. Improper integrals. (*) Hints on Multiple Integration.
- Differential equations. Part II: ODEs of first order. General principles, initial value problems, Cauchy Theorem. Qualitative sketch of solutions. Solving separable differential equation. Problem solving: modeling and understanding real world phenomena by ODEs.

LINEAR ALGEBRA

Matrix operations. Determinants and Rank. Gauss elimination method. Inverse matrix. Linear systems. Vector spaces and subspaces. Independence, basis and dimension. Linear maps from R_n to R_m : nullspace and row space. Fundamental Theorem of linear algebra on the dimensions. Linear systems theory: Cramer's rule, Rouché-Capelli Theorem. Eigenvalues and eigenvectors. Diagonalization and Spectral Theorem.

Teaching Methods

Lectures and exercises sessions by the teacher. Some topics will be introduced by using Innovative Teaching through Blended Learning sessions, MOOCs and short videos.

Verification of learning

The exam consists of a written test covering the entire course program. The test contains concept-check questions requiring an extended answer and some exercises. It is a faculty of the teacher to make sample oral tests to confirm the proposed positive grades.

The exam can be taken in each of the scheduled sessions in January-February, July and September.

At the beginning of the year, the possibility of taking an intermediate test in November-December will be discussed with the class.

Texts

- *James Stewart, Troy Day, Biocalculus. Calculus for the Life Sciences*
- *James Stewart, Calculus*
- **MOOC on Ordinary Differential Equations** www.pok.polimi.it